1 Proofs

Lemma 1. *Let $\langle N, \Pi, \tau, V \rangle$ be an MPOU game. Let $i \in N$ and $S \subset T \subseteq N \backslash \{i\}$ and $j \in T \backslash S$.*

$$v(S \cup \{i\}) - v(S) \leqslant v(S \cup \{i, j\}) - v(S \cup \{j\}) \quad (1)$$

Proof. We let $A^*(S)$ denote the assignment of profiles that maximizes the social welfare of S. In the case where there are multiple social welfare-maximizing configurations of S, we use the one with highest aggregate variance. We observe that $v(T, A^*(S)) \leqslant v(T)$ because $A^*(S)$ imposes a constraint on the behavior of S. For technical reasons, we break the proof into two cases based on whether it is more beneficial for i) i to join coalition S when S is configured to maximize $v(S \cup \{i\})$ or ii) i to join coalition $S \cup \{j\}$ when $S \cup \{j\}$ is configured to maximize $v(S \cup \{j\})$.

Case 1. $v(S \cup \{i\}) - v(S, A^*(S \cup \{i\})) > v(S \cup \{i, j\}, A^*(S \cup \{j\})) - v(S \cup \{j\})$

This inequality implies that $\sigma(S \cup \{j\}, A^*(S \cup \{j\}) < \sigma(S, A^*(S \cup \{i\}))$. Since j contributes a non-negative amount of variance, $\sigma(S, A^*(S \cup \{j\})) \leqslant \sigma(S \cup \{j\}, A^*(S \cup \{j\}))$, and likewise, $\sigma(S, A^*(S \cup \{i\})) \leqslant \sigma(S \cup \{i\}, A^*(S \cup \{i\}))$. Applying these inequalities yields $\sigma(S, A^*(S \cup \{j\})) < \sigma(S \cup \{i\}, A^*(S \cup \{i\}))$, implying:

$$\begin{aligned} &v(S \cup \{j\}) - v(S, A^*(S \cup \{j\})) < \\ &v(S \cup \{i, j\}, A^*(S \cup \{i\})) - v(S \cup \{i\}) \end{aligned} \quad (2)$$

Then, applying the inequalities $v(S, A^*(S \cup \{j\})) \leqslant v(S)$ and $v(S \cup \{i, j\}, A^*(S \cup \{i\})) \leqslant v(S \cup \{i, j\})$, and rearranging terms:

$$v(S \cup \{i\}) - v(S) < v(S \cup \{i, j\}) - v(S \cup \{j\}) \quad (3)$$

which is a stronger version of the lemma.

Case 2. $v(S \cup \{i\}) - v(S, A^*(S \cup \{i\})) \leqslant v(S \cup \{i, j\}, A^*(S \cup \{j\})) - v(S \cup \{j\})$

Applying the inequality $v(S, A^*(S \cup \{i\})) \leqslant v(S)$ on the left side yields:

$$v(S \cup \{i\}) - v(S) \leqslant v(S \cup \{i, j\}, A^*(S \cup \{j\})) - v(S \cup \{j\}) \quad (4)$$

Applying on the right side $v(S \cup \{i, j\}, A^*(S \cup \{j\})) \leqslant v(S \cup \{i, j\})$ yields the lemma:

$$v(S \cup \{i\}) - v(S) \leqslant v(S \cup \{i, j\}) - v(S \cup \{j\}) \quad (5)$$

□

Theorem 1. *The ex-ante MPOU game is convex.*

Proof. If $S = T$, then $v(S \cup \{i\}) - v(S) = v(T \cup \{i\}) - v(T)$ since the welfare-maximizing configurations of S and T are the same. If $S \subset T$, we repeatedly apply Lemma 1 to "grow" S one agent a time, creating a series of inequalities, until we relate S and T. □

Theorem 2. *Let i be an agent with two profiles π_0 and π_1 and let $A(i) = \pi_0$. Then, w.l.o.g., $D_i(x_i) = \mathcal{N}(x_i; \mu_0, \sigma_0) - \mathcal{N}(x_i; \mu_1, \sigma_1)$ is a separating function for i under A.*

Proof. We will show that the minimum of $\mathbb{E}_{x \sim \mathcal{N}(\mu_0, \sigma_0)}[\mathcal{N}(x; \mu_0, \sigma_0) - \mathcal{N}(x; \mu_1, \sigma_1)] - \mathbb{E}_{x \sim \mathcal{N}(\mu_1, \sigma_1)}[\mathcal{N}(x; \mu_0, \sigma_0) - \mathcal{N}(x; \mu_1, \sigma_1)]$ occurs when $\mu_1 = \mu_0$ and $\sigma_1 = \sigma_0$ and that the value of the expression at that point is positive.

We make use of the fact that $\mathcal{N}(x; \mu_1, \sigma_1)\mathcal{N}(x; \mu_2, \sigma_2)$ is a function proportional to the PDF of a normal distribution. Specifically,

$$\begin{aligned} &\mathcal{N}(x; \mu_0, \sigma_0)\mathcal{N}(x; \mu_1, \sigma_1) = \\ &\mathcal{N}\left(\mu_0; \mu_1, \sqrt{\sigma_0^2 + \sigma_1^2}\right) \mathcal{N}\left(x; \frac{\sigma_0^{-2}\mu_0 + \sigma_1^{-2}\mu_1}{\sigma_0^{-2} + \sigma_1^{-2}}, \frac{\sigma_0^2\sigma_1^2}{\sigma_0^2 + \sigma_1^2}\right) \end{aligned} \quad (6)$$

Then, by expanding terms and applying Equation 6:

$$\begin{aligned} &\mathbb{E}_{x \sim \mathcal{N}(\mu_0, \sigma_0)}[\mathcal{N}(x; \mu_0, \sigma_0) - \mathcal{N}(x; \mu_1, \sigma_1)] - \\ &\mathbb{E}_{x \sim \mathcal{N}(\mu_1, \sigma_1)}[\mathcal{N}(x; \mu_0, \sigma_0) - \mathcal{N}(x; \mu_1, \sigma_1)] \\ &= \frac{1}{2\sigma_0\sqrt{\pi}} - 2\mathcal{N}\left(\mu_1; \mu_0, \sqrt{\sigma_0^2 + \sigma_1^2}\right) + \frac{1}{2\sigma_1\sqrt{\pi}} \end{aligned} \quad (7)$$

We then minimize with respect to μ_1 and σ_1. Since the middle term is the only one that contains μ_1, we can minimize it separately:

$$-\frac{2}{\sqrt{2\pi(\sigma_0^2 + \sigma_1^2)}} \exp\left(-\frac{(\mu_0 - \mu_1)^2}{2(\sigma_0^2 + \sigma_1^2)}\right) \quad (8)$$

Since the argument of the exponent is always non-positive, it is maximized when it is zero, i.e., $\mu_1 = \mu_0$. Thus, we can make this substitution and rewrite the overall expression:

$$\frac{1}{2\sigma_0\sqrt{\pi}} - \frac{2}{\sqrt{2\pi(\sigma_0^2 + \sigma_1^2)}} + \frac{1}{2\sigma_1\sqrt{\pi}} \quad (9)$$

Taking the derivative with respect to σ_1^2 and setting it to zero yields two real roots of $\sigma_0 = \pm\sigma_1$. The second derivative at these points is positive. Thus, it is a minimum.

The value of the original expression at this point is 0 and positive otherwise. □

Theorem 3. *Let i be an agent with profiles Π_i and let A assign a profile to i. There exists $\boldsymbol{y}_i \in \mathbb{R}^{|\Pi_i|}$ that makes $D_i(x_i, \boldsymbol{y}_i)$ a separating function if and only if there is a linear combination of the difference vectors of $D_i(x_i, \boldsymbol{y}_i)$ that has only positive entries.*

Proof. First, we prove the forward direction. Let $\boldsymbol{c}$ be the coefficients of the linear combination of the difference vectors that has only positive entries, i.e., $\sum_{k \in |\Pi_i|} \boldsymbol{c}_k \boldsymbol{d}_k = \boldsymbol{b}$ where $\boldsymbol{b}$ is element-wise positive. Then, $\mathbb{E}_{x_i \sim A(i)}[D_i(x_i, \boldsymbol{c})] - \mathbb{E}_{x_i \sim \pi}[D_i(x_i, \boldsymbol{c})] = \boldsymbol{c}\boldsymbol{d}_k = \boldsymbol{b}_{k-1}$. Since $\boldsymbol{b}$ is element-wise positive, letting $\boldsymbol{y}_i = \boldsymbol{c}$ makes $D_i(x_i, \boldsymbol{y}_i)$ a separating function.

The reverse direction is also straightforward. Suppose $D_i(x_i, \boldsymbol{y}_i)$ is a separating function. Then, let $\boldsymbol{b}_{k-1} = \mathbb{E}_{x_i \sim A(i)}[D_i(x_i, \boldsymbol{c})] - \mathbb{E}_{x_i \sim \pi}[D_i(x_i, \boldsymbol{c})] = \boldsymbol{y}_i \cdot \boldsymbol{d}_k$. Thus, taking $\boldsymbol{y}_i$ as the coefficients of the linear combination of difference vectors equals $\boldsymbol{b}$, which has only positive entries. □

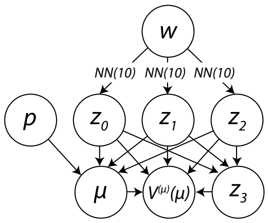

Figure 1: The form of the learned valuation model. $NN(10)$ denotes a neural network with 10 hidden units.

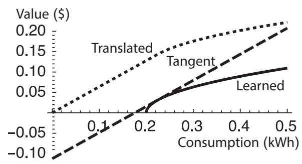

Figure 2: Translating the valuation function to pass through the origin.

Corollary 1. *Let $\boldsymbol{d}_k$ be the differences vectors for agent i. If the difference vectors are linearly independent, a setting of $\boldsymbol{y}_i$ exists that makes $D_i(x_i, \boldsymbol{y}_i)$ a separating function.*

Proof. If the difference vectors are linearly independent, there exists a coefficient vector $\boldsymbol{c}$ that makes $\sum_{k \in |\Pi_i|} \boldsymbol{c}_k \boldsymbol{d}_k$ elementwise positive. We can take $\boldsymbol{y}_i = \boldsymbol{c}$. □

2 Model Details

As described in the in paper, agent i's utility $V_i(w, \mu, \sigma)$ is decomposed into $V_i(w, \mu, \sigma) = V_i^{(\mu)}(w, \mu) V_i^{(\sigma)}(\sigma, \mu)$. Then $V_i^{(\mu)}(w, \mu)$ is restricted to the form:

$$V_i^{(\mu)}(w, \mu) = z_i^{(0)}(w) \left(\mu - z_i^{(1)}(w)\right)^{z_i^{(2)}(w)} + z_i^{(3)}(w) \tag{10}$$

constraining $z_i^{(0)} > 0$, $z_i^{(1)} > 0$, $0 < z_i^{(2)} < 1$, $z_i^{(3)}(w) \geqslant 0$ (Figure 1 depicts the utility model). The term $z_i^{(3)}(w)$ has no influence on predictions: it can be viewed as inherent value due to weather, and is used to account for the flexibility provided by the $z_i^{(1)}$ term, which may create valuations where consumption 0 yields negative value (violating our assumptions). To prevent this, we set $z_i^{(3)}(w)$ to ensure the tangent at the predicted consumption for \$0.64 (the largest price in the data set) passes through (0,0). (see Figure 2). When this tangent crosses the y-axis above 0, we set $z_i^{(3)}(w) = 0$ and splice in an exponential function of the form ax^b that passes through (0,0) and matches the derivative at the splicing point.

Figure 4 shows the learned valuation functions for the 25 households. Each line represents a household's response to different weather conditions.

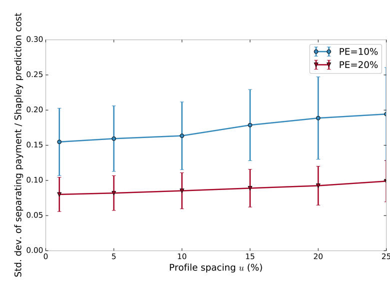

Figure 3: Comparison of the standard deviation of the separating function payment to the ex-ante payment for prediction accuracy. Bars show one standard deviation. 5000 agents, 100 trials.

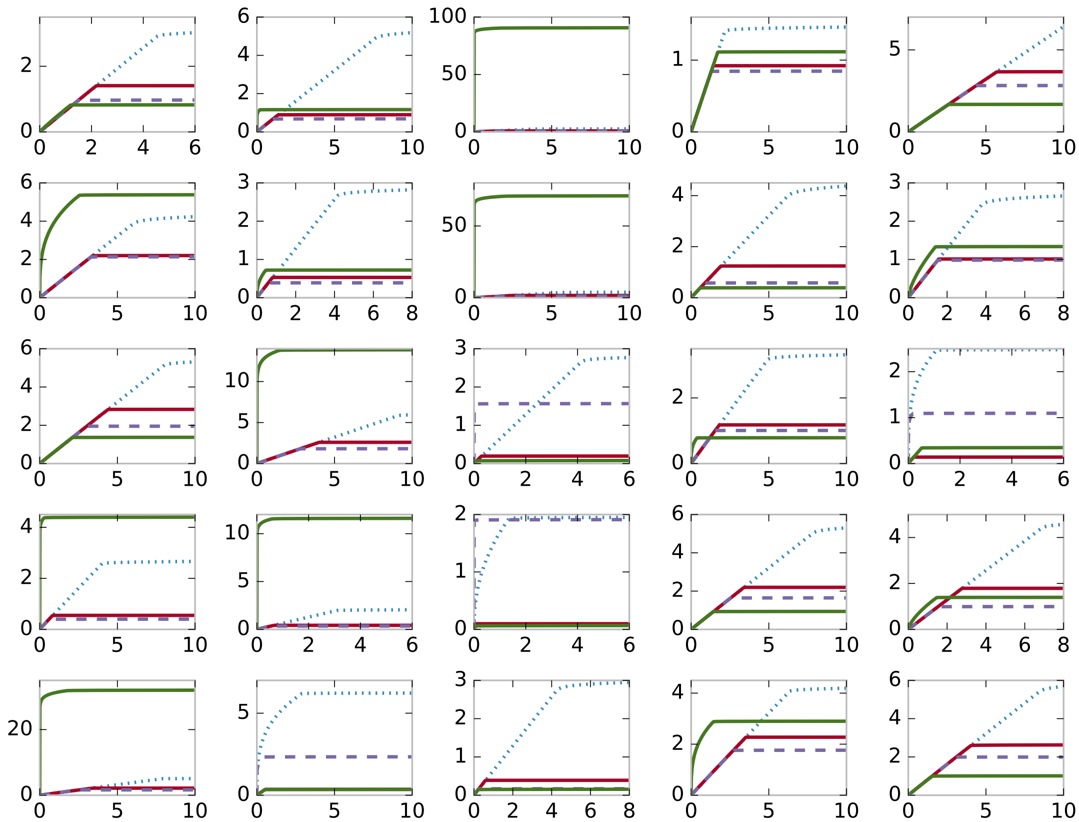

Figure 4: Learned value models for the 25 households with consumption mean (kwh) on the x-axis and value ($) on the y-axis. The red line represents the median weather conditions. The dotted line represents the median day with 90th percentile or higher temperature. The dashed and green lines are the same for sunshine and humidity, respectively.